Name: C - Key

Date:

Q4 Review Sheet

PreCalculus

Evaluate each limit.

1. $\lim_{x\to 2}(x^2-4x+1)$ $= 4-8+1 = \boxed{-3}$

2. $\lim_{x\to 0}(\frac{1}{3^x})$ $= \boxed{1}$

3. $\lim_{x\to 0}\frac{(9+x)^2-81}{x}$ $81+18x+x^2$ $= \boxed{18}$

4. $\lim_{x\to -1}\frac{x+1}{x+3}$ $= \frac{0}{2} = \boxed{0}$

5. $\lim_{x\to -3}\frac{2x-6}{|x-3|}$ $= \frac{2(x-3)}{x-3}, \frac{2(x-3)}{-(x-3)}$ $2, -2$ $= \boxed{-2}$

6. $\lim_{x\to 10}\frac{x^2-100}{x-10}$ $= x+10 = \boxed{20}$

7. $\lim_{x\to 64}\frac{\sqrt{x}-8}{64-x}$ $\frac{\sqrt{x}+8}{\sqrt{x}+8}$ $x-64$ $= \frac{-1}{\sqrt{x}+8} = \boxed{-\frac{1}{16}}$

8. $\lim_{x\to -1}\sqrt{x^2-1}$ $= \boxed{0}$

9. $\lim_{x\to 0}\frac{1}{x}$ $= \boxed{DNE}$

10. $\lim_{x\to -1}\frac{x^2+8x+7}{x^2+5x+4}$ $\frac{(x+1)(x+7)}{(x+1)(x+4)}$ $= \frac{6}{3} = \boxed{2}$

11. $\lim_{x\to 6}\frac{36-x}{\sqrt{x}-6}$ $\frac{\sqrt{x}+6}{\sqrt{x}+6}$ $x-36$ $= \boxed{-\sqrt{6}-6}$

12. $\lim_{x\to -1}\frac{1+x}{\sqrt{x+1}}$ $\frac{\sqrt{x+1}}{\sqrt{x+1}}$ $= \boxed{0}$

13. $\lim_{x\to 13^+}\frac{|x-1|}{x+1}$ $\frac{x-1}{x+1} = \frac{13-1}{13+1} = \frac{12}{14} = \boxed{\frac{6}{7}}$

14. $\lim_{x\to 2}\frac{-2}{(x-2)^2}$ $= \boxed{-\infty}$

15. $\lim_{x\to 5}\frac{\frac{1}{x}-\frac{1}{5}}{5-x}$ $5x = \frac{5-x}{(5-x)5x} = \boxed{\frac{1}{25}}$

16. $\lim_{x\to 0}\frac{12^{-1}-(12+x)^{-1}}{12x(12+x)}$ $\frac{1}{12}-\frac{1}{12+x}$ $12+x-12$ $12(12+x)$ $\frac{1}{12(12+x)} = \boxed{\frac{1}{144}}$

17. $\lim_{x\to 0}\frac{x}{(15+x)^2-225}$ $\boxed{\frac{1}{30}}$

18. $\lim_{x\to\infty}\frac{x^3}{x^2-1}$ $= \boxed{\infty}$

19. $\lim_{x\to\infty}\frac{x^3+x^4}{x^6-1}$ $= \boxed{0}$

20. $\lim_{x\to -\infty}\frac{2x^3}{x^3-1}$ $= \boxed{\frac{2}{1}}$

21. $\lim_{x\to -\infty}\frac{4-x}{x^3+1}$ $= \boxed{0}$

22. $\lim_{x\to -\infty}(x-3)^{-2}$ $= \boxed{0}$ $\frac{1}{(x-3)^2}$

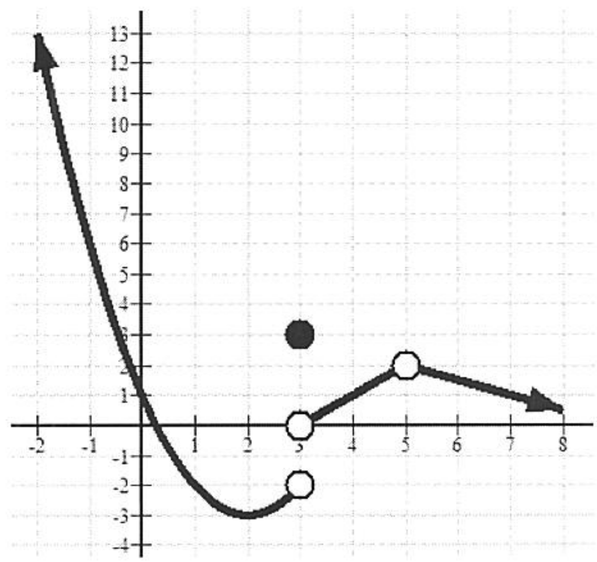
23. $\lim_{x\to\infty}\frac{x-3}{x^2-9}$ $= \frac{1}{x+3} = \boxed{0}$

24. $\lim_{x\to -3}\frac{x-3}{x^2-9}$ $= \boxed{DNE}$

25. $\lim_{x\to 3}\frac{x-3}{x^2-9}$ $= \boxed{\frac{1}{6}}$

26. $\lim_{x\to\infty}5-\frac{4}{x^4}$ $= \boxed{5}$

For questions 27 – 35, use the graph of f(x) provided below.




27. $\lim_{x\to\infty} f(x)$ $-\infty$

28. $\lim_{x\to 3^-} f(x)$ $-2$

29. $\lim_{x\to 3} f(x)$ DNE

30. $\lim_{x\to 5^-} f(x)$ 2

31. $\lim_{x\to 5} f(x)$ 2

32. $\lim_{x\to -\infty} f(x)$ $\infty$

33. $\lim_{x\to 3^+} f(x)$ 0

34. $f(3)$ 3

35. $\lim_{x\to 5^+} f(x)$ 2

36. $f(5)$ $\varnothing$

37. Evaluate: $\lim_{x \to 6} f(x)$ , $f(x) = \begin{cases} x^2 - 36 = 0 & , x > 6 \\ x^2 - 14x + 48 & , x \le 6 \end{cases}$ $= \boxed{0}$

$36 - 14(6) + 48 = 36 - 84 + 48 = 0$

Determine whether each of the following is a parabola, hyperbola, ellipse, circle or none. (38-42)

38. $y^2 + 2x^2 - 4 = 0$ Ellipse
39. $y + 2x^2 - 4 = 0$ parabola
40. $y + 2x - 4 = 0$ none
41. $2y^2 + 2x^2 - 4 = 0$ circle
42. $y^2 - 2x^2 - 4 = 0$ hyperbola.

Write the equation of each in standard form and determine which type of conic section the equation represents.(ellipse, circle, or hyperbola) (43-45)

43. $y^2 + x^2 + 2x = 0$ $y^2 + x^2 + 2x + 1 = 1$ $y^2 + (x+1)^2 = 1$ (circle)
44. $y^2 + 4y + 2x^2 - 4x = -2$ $y^2 + 4y + 4 + 2(x^2 - 2x + 1) = -2 + 4 + 2$ $(y+2)^2 + 2(x-1)^2 = 4$ $\frac{(y+2)^2}{4} + \frac{(x-1)^2}{2} = 1$ (ellipse)
45. $4y^2 + 32y - x^2 - 16x - 16 = 0$

$-x^2 - 16x + 4y^2 + 32y = 16$ $-1(x^2 + 16x + 64) + 4(y^2 + 8y + 16) = 16 - 64 + 64$ $-(x+8)^2 + 4(y+4)^2 = 16$

$-\frac{(x+8)^2}{16} + \frac{(y+4)^2}{4} = 1$ $\frac{(y+4)^2}{4} - \frac{(x+8)^2}{16} = 1$ (hyperbola)

46. Find the center and radius of $y^2 + x^2 - 2x = 0$.

$x^2 - 2x + y^2 = 0$ $x^2 - 2x + 1 + y^2 = 0 + 1$ $(x-1)^2 + y^2 = 1$

$c = 1, 0$ radius $= 1$

Find the center, foci, and vertices of each of the following.

47. $\frac{x^2}{81} + \frac{(y+1)^2}{144} = 1$

$a^2 = 144$ $a = 12$
$b^2 = 81$ $b = 9$
$c^2 = 63$ $c = \sqrt{63}$

Center: $(0, -1)$
Vertices: $(0, 11)$ $(9, -1)$ $(0, -13)$ $(-9, -1)$
Foci: $(0, -1 + \sqrt{63})$ $(0, -1 - \sqrt{63})$

48. $\frac{(y-1)^2}{4} - \frac{(x+4)^2}{36} = 1$

$a^2 = 4$ $a = 2$
$b^2 = 36$ $b = 6$
$c^2 = 40$ $c = \sqrt{40}$

Center: $(-4, 1)$
Vertices: $(-4, 3)$ $(-4, -1)$
Foci: $(-4, 1 + \sqrt{40})$ $(-4, 1 - \sqrt{40})$

Sketch/Graph each of the following (51-54). For each, state the coordinates of the center, foci, and vertices. In addition, for hyperbolas include the equations of the asymptotes and state whether the transverse axis is vertical or horizontal.

49. $x^2 - \frac{(y-1)^2}{9} = 1$

On Graph last page

50. $\frac{(y-3)^2}{4} + \frac{(x+1)^2}{9} = 1$

51. Write the equation of the circle whose diameter has the endpoints (-1,5) and (3, - 5)?

Center (use midpoint formula)

$\frac{-1+3}{2}, \frac{5+-5}{2}$

$\frac{2}{2}, \frac{0}{2}$

$(1, 0)$

Radius (use distance formula)

$r = \frac{1}{2}\sqrt{(-1-3)^2 + (5--5)^2}$
$= \frac{1}{2}\sqrt{(-4)^2 + (10)^2}$
$= \frac{1}{2}\sqrt{16 + 100}$
$= \frac{1}{2}\sqrt{116}$
$= \frac{1}{2} 2\sqrt{29} = \sqrt{29}$

$\boxed{(x-1)^2 + (y)^2 = 29}$

52. A manufacturer wants to maximize the profit for two products. Product 1 yields a profit of \$2.00 per unit and product 2 yields a profit of \$2.50 per unit. Market tests and available resources have indicated the following:

a. The combined production level should not exceed 1200 units per month.
b. The demand for product 2 is no more than half the demand for product 1.
c. The production level of product 1 is less than or equal to 600 units plus three times the production level of product 2.

Find the number of units of each product the manufacturer should produce to maximize the profit.

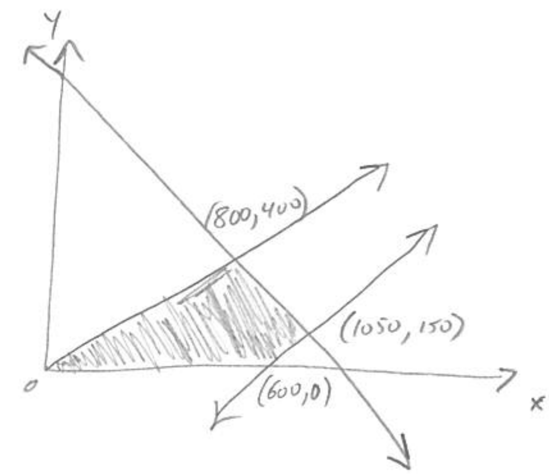
Let x = # of Product 1
y = # of Product 2

Constraints

$x + y \le 1200 \longrightarrow y \le -x + 1200$

$y \le \frac{1}{2}x \longrightarrow y \le \frac{1}{2}x$

$x \le 3y + 600 \longrightarrow y \ge \frac{1}{3}x - 200$




Objective Function

Profit = $2x + 2.5y$

Vertices

| | |
|---|---|
| $(0,0)$ | $2(0) + 2.5(0) = 0$ |
| $(600,0)$ | $2(600) + 2.5(0) = 1200$ |
| $(800,400)$ | $2(800) + 2.5(400) = 2600$ |
| $(1050,150)$ | $2(1050) + 2.5(150) = 2475$ |

800 of product #1
400 of product #2
for a maximum profit of \$2600.

53. An accounting firm has 800 hours of staff time and 90 hours of reviewing time available each week. The firm charges $1500 for an audit and $250 for a tax return. Each audit requires 50 hours of staff time and 5 hours of review time. Each tax return requires 10 hours of staff time and 2 hours of review time. What numbers of audits and tax returns will yield the maximum revenue?

Let $x$ = # of audits

$y$ = # of tax returns

Constraints

$50x + 10y \le 800$

$5x + 2y \le 90$

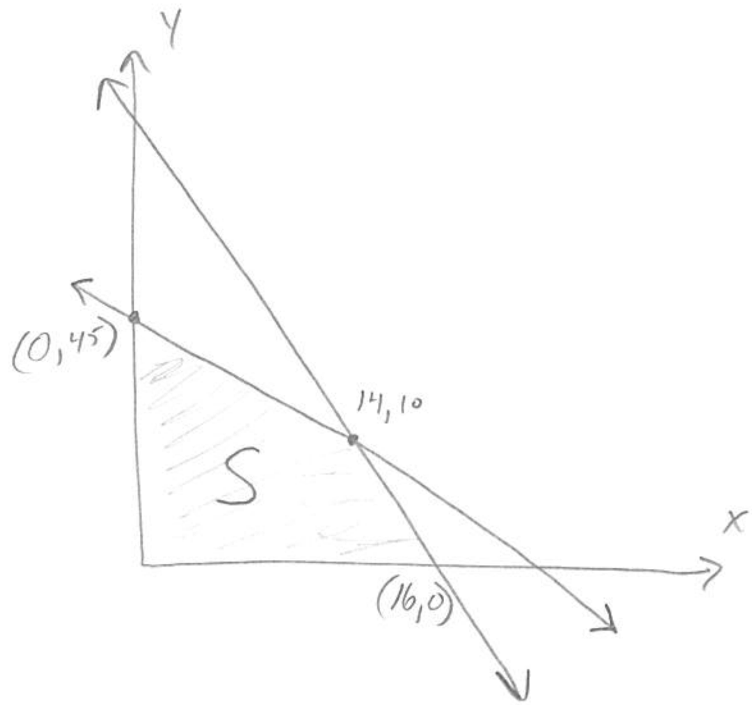
Objective

$1500x + 250y = \text{Revenue}$

$(0,45)$ $1500(0) + 250(45) = 11250$

$(14,10)$ $1500(14) + 250(10) = 23500$

$(16,0)$ $1500(16) + 250(0) = 24000$

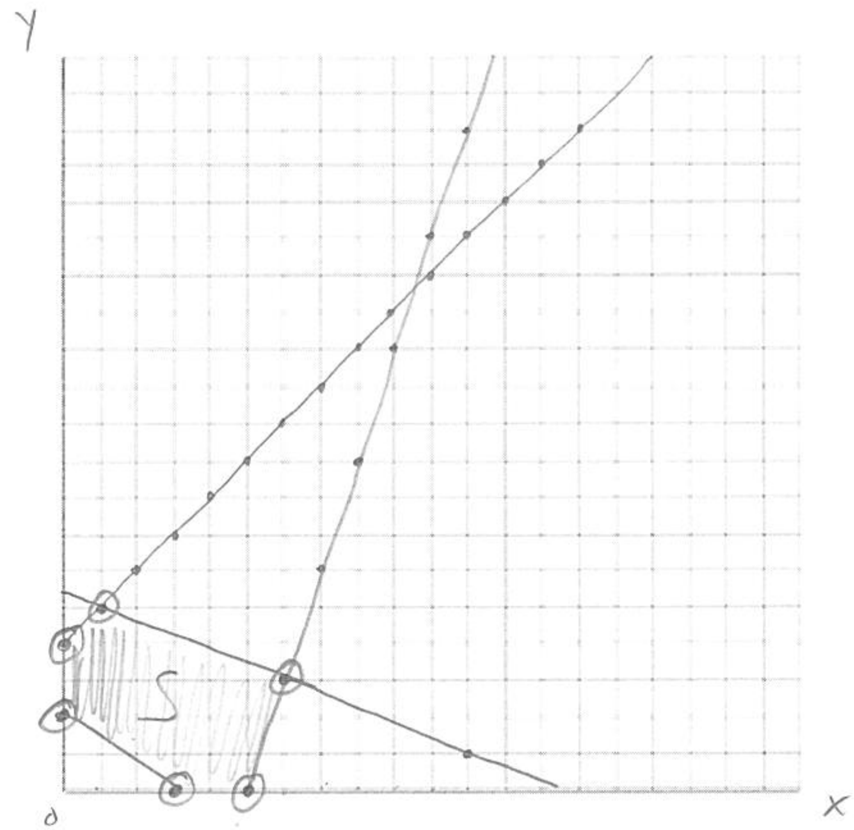



16 audits and 0 reviews

for revenue of $24,000

54. Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given objective function for this region.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + 3y \geq 6 \\ 3x - y \leq 15 \\ -x + y \leq 4 \\ 2x + 5y \leq 27 \end{cases}$$




$$z = 5x + 7y$$

$y \geq -\frac{2}{3}x + 2$

$y \geq 3x - 15$

$y \leq x + 4$

$y \leq -\frac{2}{5}x + \frac{27}{5}$

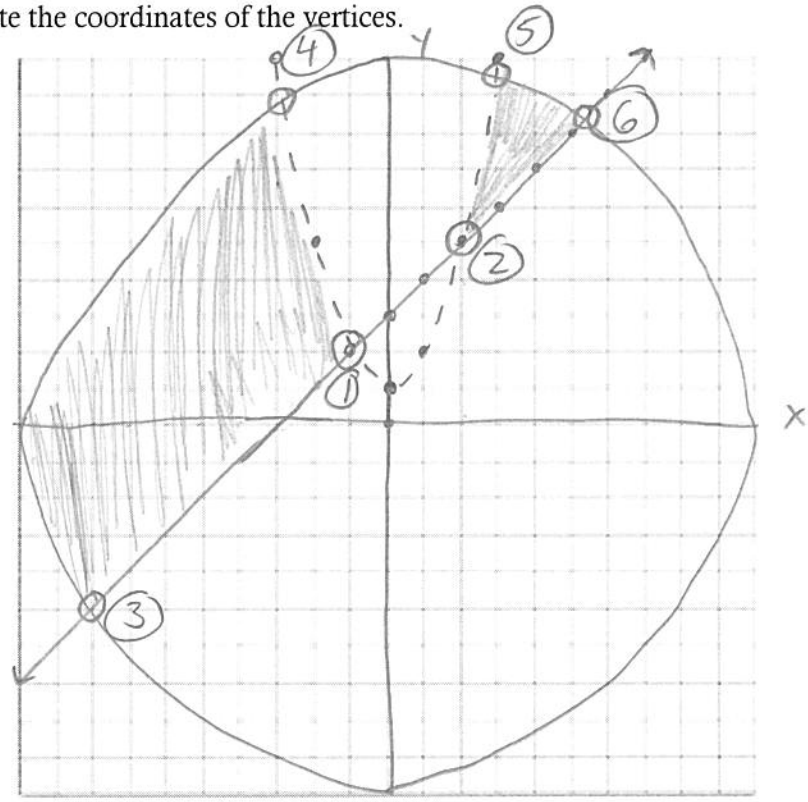
| | |
|---|---|
| $(0,2)$ | $5(0) + 7(2) = 14$ |
| $(3,0)$ | $5(3) + 7(0) = 15$ |
| $(5,0)$ | $5(5) + 7(0) = 25$ |
| $(6,3)$ | $5(6) + 7(3) = 51$ |
| $(1,5)$ | $5(1) + 7(5) = 40$ |
| $(0,4)$ | $5(0) + 7(4) = 28$ |

Min of 14 @ $(0,2)$

Max of 51 @ $(6,3)$

55. Graph the system of inequalities. State the coordinates of the vertices.

$$\begin{cases} x^2 + y^2 \le 100 \\ x^2 - y > -1 \\ y \ge x + 3 \end{cases}$$




vertices:

(-1, 2) ①

(2, 5) ②

(-8.410, -5.410) ③

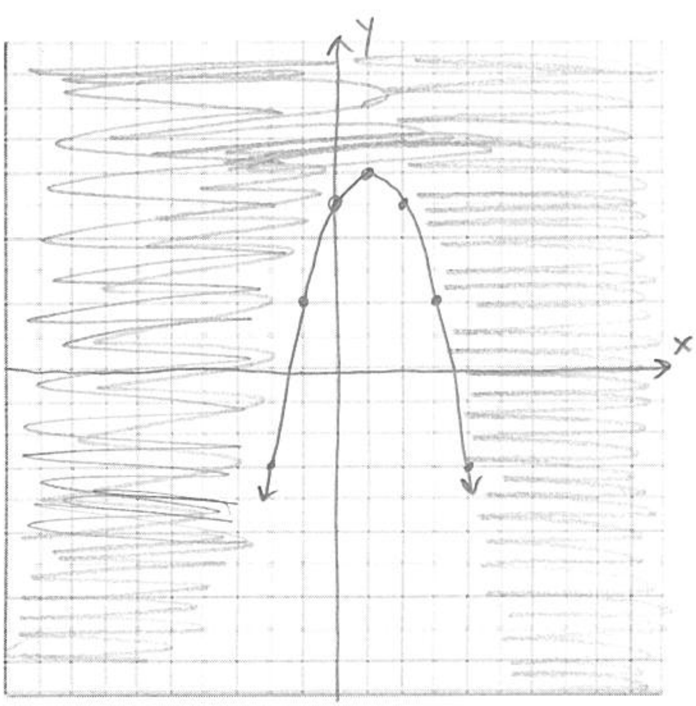
(-2.926, 9.562) ④

(2.926, 9.562) ⑤

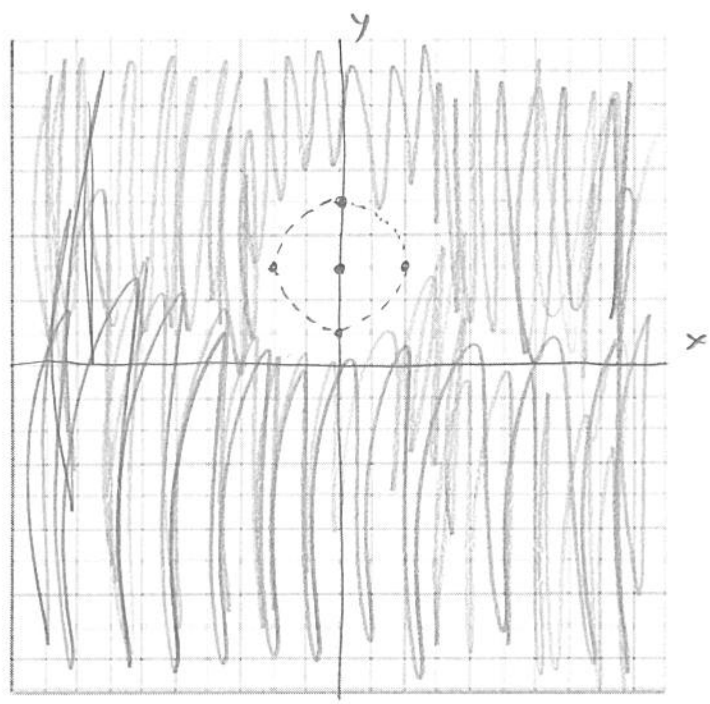
(5.410, 8.410) ⑥

56. Sketch the solution to the inequalities on the graph paper provided.

a. $y - 5 \ge -x^2 + 2x$




b. $x^2 + (y-3)^2 > 4$



49) $\dfrac{x^2}{1} - \dfrac{(y-1)^2}{9} = 1$

$a^2 = 1 \quad a = 1$

$b^2 = 9 \quad b = 3$

$c^2 = 10 \quad c = \sqrt{10}$

Hyperbola

Asymptotes:

$(y-1) = \pm \frac{3}{1}(x)$

Center

$(0,1)$

Vertices

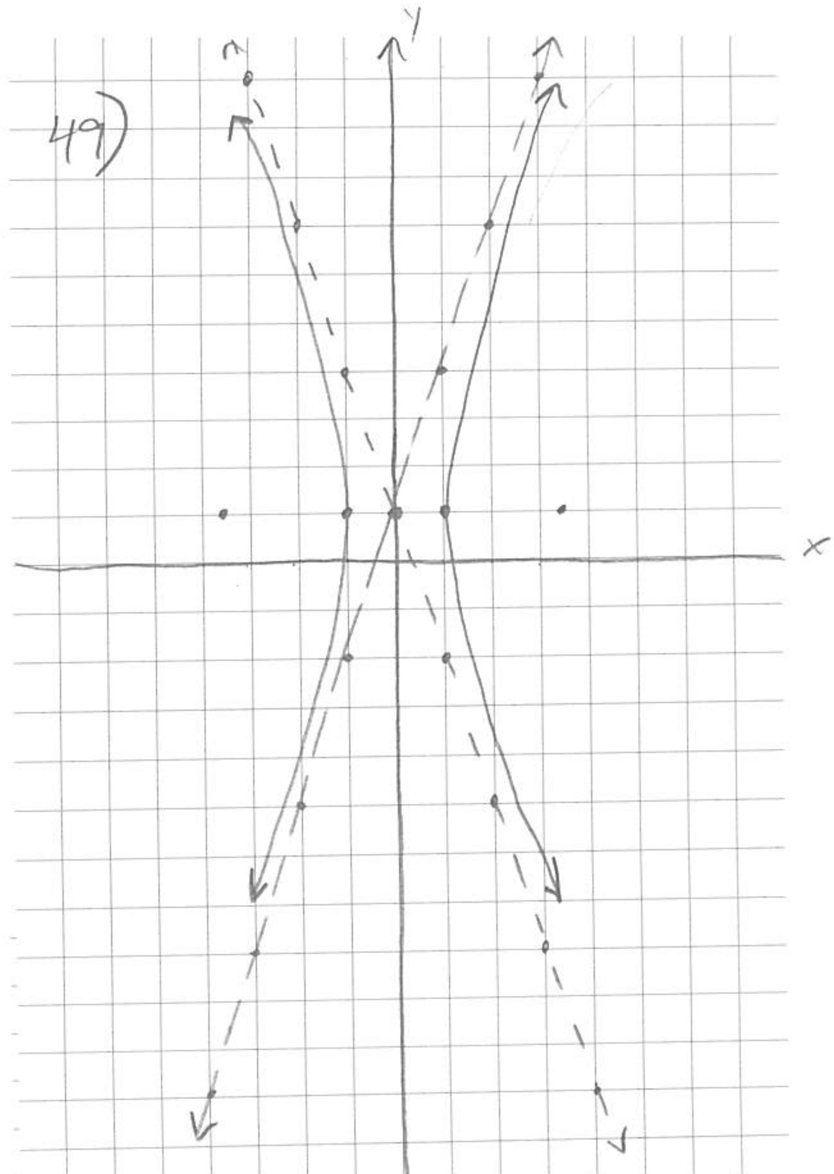
$(1,1)$

$(-1,1)$

Foci

$(\sqrt{10}, 1)$

$(-\sqrt{10}, 1)$




50) $\dfrac{(y-3)^2}{4} + \dfrac{(x+1)^2}{9} = 1$

$a^2 = 9 \quad a = 3$

$b^2 = 4 \quad b = 2$

$c^2 = 5 \quad c = \sqrt{5}$

Ellipse

Center:

$(-1,3)$

Vertices

$(2,3)$

$(-4,3)$

$(-1,5)$

$(-1,1)$

Foci

$(-1+\sqrt{5}, 3)$

$(-1-\sqrt{5}, 3)$